# STATUS AND FUTURE OF DETERMINATION OF AERODYNAMIC DERIVATIVES FROM FLIGHT DATA

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## ABSTRACT

The evaluation of aerodynamic derivatives from flight data based on system identification is considered. The estimation procedure includes the equation error method, the output error method and the generalized maximum likelihood method. The problems concerning accuracy, sensitivity and identifiability are also discussed. The general computing algorithm for the first two methods and the future development in the area of aircraft parameter estimation are briefly mentioned.

The maximum likelihood estimation technique is demonstrated in two examples. The first example includes the longitudinal short period motion of a slender delta-wing research aircraft, the second one the lateral motion of a fighter aircraft.

## 1. INTRODUCTION

The aerodynamic derivatives of an aircraft are the partial derivatives of aerodynamic forces and moments acting on the aircraft with respect to its state variables (the stability derivatives) and input variables (the control derivatives). The values of these derivatives can be obtained from theoretical calculations, wind-tunnel test and in-flight measurement.

Previous approaches to the evaluation of aerodynamic derivatives from flight data were based mainly on time consuming steady-state measurements and on the measurement of free oscillations. The analysis of transient manoeuvres based upon the least squares procedure was firstly proposed by Greenberg [(1)] and Shinbrot [(2)]. It was, however, applied to very simple manoeuvres and it resulted in only limited information about system parameters and their accuracies.

For the practical analysis of more complicated manoeuvres the analog-matching technique has been used. This technique minimizes the errors of the various responses iteratively through the human operation.

Finally the increased availability of modern digital computers made the application of more sophisticated techniques for the estimation of aircraft parameters (and thus of non-dimensional aerodynamic derivatives) feasible. These new techniques are part of a general strategy and process called identification which can establish the properties of any system by the measurement of its input and output time histories.

The identification of an aircraft using modern control theory, theory of statistical inference and new digital techniques has brought qualitatively new ways of aircraft testing and flight data analysis. This approach enables us to evaluate from one test run all the stability and control derivatives involved or their combinations, together with their accuracies and confidence intervals. At the same time the accuracy of measured data is also estimated so that this data can be used in the analysis with a corresponding level of confidence.

If necessary, there is the possibility of separating the measurement noise in the output variables from the external disturbances to the system caused by gust effects or modelling errors (process noise).

The identification techniques give the opportunity of including in the analysis the a priori knowledge of aircraft parameters obtained from wind tunnel measurements or previous flight measurement.

Finally the identification methods provide tools for a proper design of an experiment (e.g. optimal input form) to obtain the most accurate results and for testing the hypothesis about the correct form of the mathematical model describing the analysed motion of an aircraft.

Substantial contributions in the field of aircraft identification were made by several authors (3) - (7) and by the NASA as an organisation.

At the Cranfield Institute of Technology (C.I.T) the research in aircraft identification was initiated about three years ago. Some of the results achieved are presented in [(8)].

## 2. IDENTIFICATION

Zadeh defines the identification as "Identification is the determination, on the basis of input and output, of a system within a specified class of systems, to which the system under test is equivalent." (9)

Using this formulation it is necessary to specify a class of systems, a class of input signals and the meaning of "equivalent," and then to essemble the a priori knowledge on the structure of the mathematical model for the system under test and measurements of the input and output variables. The identification procedure involves generally three steps, namely characterization, parameter and state estimation and verification.

### 2.1 Characterization

Characterization is a qualitative operation defining the structure of a system. In the case of aircraft identification the system is usually assumed to be deterministic with time invariant parameters described by the set of linear or non-linear differential equations. The unknown parameters include, in general, the coefficients of equations of motion, initial conditions and constant bias terms in measured output variables. The measured variables are used in the form of sampled time histories with a constant sampling rate.

To represent any realistic flying vehicle completely would be a task of immence difficulty. The problem is to select the simplest approximate representation that will permit the successful determination of the unknown parameters from measured data.

The linear equations describing the longitudinal and lateral motion of an aircraft are well developed and have the form

$$T\dot{x} = A'x + B'u, \quad x(0) = \alpha \qquad (1)$$

$$y = Cx + Du \qquad (2)$$

where x, u and y are the state, input and output vector, respectively, and $\alpha$ is the vector of initial conditions.

Because T is a nonsingular matrix then by letting

$$A = T^{-1}A' \text{ and } B = T^{-1}B'$$

equation (1) can be modified to the more standard form

$$\dot{x} = Ax + Bu, \quad x(0) = \alpha \qquad (3)$$

The unknown parameters $\beta$ are contained in all four matrices A, B, C and D.

The modelling of an aircraft during large disturbance manoeuvres or in extreme flight conditions is a difficult task demanding the formulation of a set of nonlinear equations

$$\dot{x} = f(x, u, \beta), \quad x(0) = \alpha \qquad (4)$$

$$y = g(x, u, \beta) \qquad (5)$$

For the new families of STOL and VTOL aircraft nonlinear forms of their models must normally be used.

The problem of modelling a complicated system raises the fundamental question of how complex the model should be. Although a more complex model can be justified for proper description of aircraft motion it is not clear in the case of parameter estimation what should be the best relationship between model complexity and measurement information. If too many unknown parameters are sought for a limited amount of data then a reduced reliability of evaluated parameters can be expected, or attempts to identify all parameters might fail.

### 2.2 Parameter and state estimation

The second step of the identification forms the parameter and state estimation. Several techniques based upon methods of estimation theory are used. An excellent review of them is given in (10).

The estimation methods minimize the optimality criterion, termed the cost function. The cost function often defines the equivalence in the Zadeh's definition mentioned and it is expressed as a functional of the system output and the model output. Two models are then said to be equivalent if the value of the cost function is the same for both of them.

Parameter estimation results in the determination of the numerical values of unknown parameters, in the estimation of their accuracy and the accuracy of the identification process. With the exception of the simple methods state estimation is included in the whole estimation process as its integral part.

### 2.3 Verification

Verification followes immediately the estimation and sometimes may be a part of a solution of identifiability problems. The purpose of verification is to relate the results obtained to well-known physical points of the system under investigation. This approach can raise the question of the reliability of estimates in general and the problem of correct mathematical modelling in particular. If some inconsistency appears the whole problem of identification is eventually reconsidered at the characterization level.

## 3. ESTIMATION METHODS

There are several methods for the estimation of aircraft parameters which are now quite well established as the part of identification process. Their basic differences are due to variety of assumptions regarding an optimal criterion, the prior probability, the appearance of external disturbances to the system and the presence of measurement noise. It is convenient to divide these methods into three groups, namely the equation error methods, the output error methods and the general maximum likelihood method.

### 3.1 The Equation Error Methods

The equation error methods represent the application of the regression analysis to each state equation separately minimizing the sum of squared errors satisfying the equation. For these methods it is, therefore, assumed that the input variables and all state variables and their derivatives can be obtained from measurement, and that only state variable derivatives are corrupted by noise.

The cost function has the form

$$J_r = \frac{1}{2}\sum_{i=1}^{N} (\dot{x}_{rEi} - \dot{x}_{ri})^2 \qquad (6)$$

$$r = 1, 2, \ldots, n$$

where n and N are the number of state equations and the number of data points, respectively. The index E denotes the measured quantity. The least squares solution is obtained by finding the minimum of J.

### 3.2 The Output Error Methods

The output error methods minimize the errors between the actual output and the model output computed by using the same input. It is assumed that only measured outputs are corrupted by noise and that there are no gust or other flight disturbances. The optimalization problem involved is a nonlinear one and requires the use of iterative methods. The modified Newton-Raphson method is usually applied because of its good convergence rate even for large number of unknown parameters.

The general scheme for the output error methods is given in Fig. 1. Regarding the assumptions about the probability distribution of measurement noise and the a priori information about the unknown parameters three estimation techniques are considered.

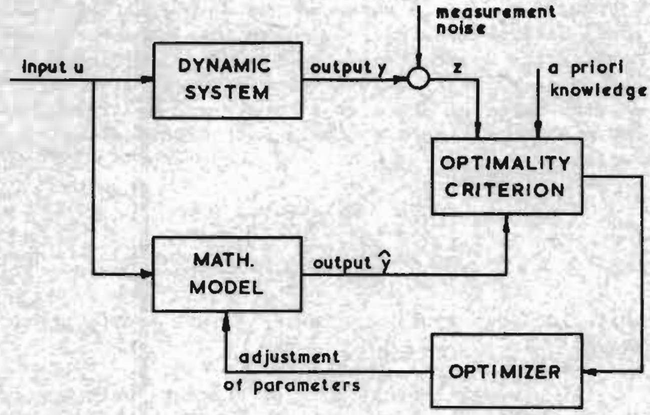


Fig.1 Parameter Estimation using Output Error Method

Weighted Least Squares (WLS) Estimation:-

The measurement equation is

$$z = y + n \tag{7}$$

and the cost function is formed as

$$J = \frac{1}{2}\sum_{i=1}^{N} (z_i - y_i)^T W_1 (z_i - y_i) \tag{8}$$

In these equations $z$ is the measurement vector, $n$ is the vector of measurement errors, $W_1$ is the weighting matrix, and $T$ denotes a transpose matrix.

The cost function is minimized with respect to the unknown parameters $\gamma$, where $\gamma^T = \{\alpha^T, \beta^T\}$. After the kth iterations the new estimate of unknown parameters is obtained as

$$\hat{\gamma}_k = \hat{\gamma}_{k-1} + \{\sum_{i=1}^{N} H_i^T W_1 H_i\}^{-1} \sum_{i=1}^{N} H_i^T W_1 (z_i - \hat{y}_i) \tag{9}$$

where $H$ is the sensitivity matrix. The jth column of $H$ is $\partial y/\partial \gamma_j$, where $\gamma_j$ is the jth element in the vector $\gamma$.

The error covariance matrix of unknown parameters is

$$E\{(\gamma-\hat{\gamma})(\gamma-\hat{\gamma})^T\} = \sigma^2 \{\sum_{i=1}^{N} H_i^T W_1 H_i\}^{-1} \tag{10}$$

where $E(\cdot)$ is the expected value and $\sigma^2$ is estimated from the weighted sum of squared residuals

$$v_i = z_i - \hat{y}_i \tag{11}$$

Maximum Likelihood (ML) Estimation:-

It is assumed for this technique that in (7) $n$ is the random gaussian vector with zero mean and covariance matrix $R_1$. The ML estimate of the unknown parameters $\gamma$ and unknown coefficients in $R_1$ can be obtained by maximizing the likelihood function

$$L(p) = -\frac{1}{2}\sum_{i=1}^{N}(z_i - y_i)^T R_1^{-1} (z_i - y_i) + - \frac{N}{2}\ln|R_1| + \text{const.} \tag{12}$$

where $|R_1|$ denotes the determinant of $R_1$.

The new estimate of unknown parameters $\gamma$ is given as

$$\hat{\gamma}_k = \hat{\gamma}_{k-1} + \{\sum_{i=1}^{N} H_i^T \hat{R}_1^{-1} H_i\}^{-1} \sum_{i=1}^{N} H_i^T \hat{R}_1^{-1} (z_i - \hat{y}_i) \tag{13}$$

while the estimate of $R_1$ is found from

$$\hat{R}_1 = \frac{1}{N}\sum_{i=1}^{N} v_i v_i^T \tag{14}$$

A lower bound on the error covariance matrix for the estimated parameters is given by the Cramèr-Rao inequality

$$E\{(\gamma-\hat{\gamma})(\gamma-\hat{\gamma})^T\} = M^{-1} \tag{15}$$

where $M$ represents the Fisher information matrix defined as

$$M = E\{(\frac{\partial L}{\partial \gamma})(\frac{\partial L}{\partial \gamma})^T\} \tag{16}$$

The estimate of the information matrix can be found from

$$M = \sum_{i=1}^{N} H_i^T \hat{R}_1^{-1} H_i \tag{17}$$

Bayesian Estimation:-

The Bayesian estimation technique uses for the parameter estimation both the information contained in measured data and the a priori information about the parameters involved. Applying Bayesian rule and treating $\gamma$ as a random gaussian vector with mean value $\gamma_o$ and covariance $R_2$, then by minimizing the posterior distribution the solution is obtained in form

$$\hat{\gamma}_k = \hat{\gamma}_{k-1} + \{\hat{R}_2^{-1} + \sum_{i=1}^{N} H_i^T \hat{R}_1^{-1} H_i\}^{-1} \times \{-\hat{R}_2^{-1}(\gamma - \gamma_o) + \sum_{i=1}^{N} H_1^T \hat{R}_1^{-1} (z_i - \hat{y}_i)\} \tag{18}$$

The estimate of $R_1$ is obtained from (14) whereas the Cramèr-Rao bound on the error covariance matrix of the estimates is given as

$$M^{-1} = \{\hat{R}_2^{-1} + \sum_{i=1}^{N} H^T \hat{R}_1^{-1} H_i\}^{-1} \tag{19}$$

## 3.3 The General Maximum Likelihood Method

The general maximum likelihood method is capable of solving the most general identification problem including the presence of additive random process noise in the equations of motion and random disturbances corrupting the measured inputs and outputs. The concept and application of this method using Kalman filter (for a linear system) and extended Kalman filter (for a nonlinear system) is presented in [5] and [6].

The scheme for the method is given in Fig. 2. The state equation of the system under test can be expressed as

$$\dot{x} = f(x, u, \beta) + \Gamma w \tag{20}$$

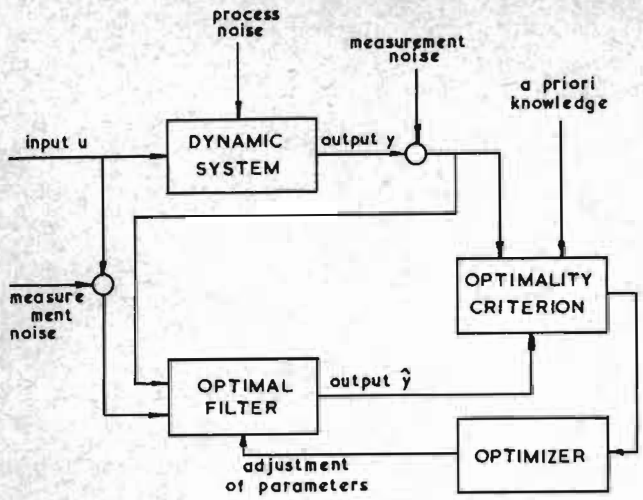


Fig.2 Parameter Estimation using Generalized Maximum Likelihood Method.

where w is the process noise vector (assumed zero mean and white with spectral density matrix Q). The optimizing criterion is the likelihood function

$$L(p) = -\frac{1}{2}\sum_{i=1}^{N}(z_i-y_i)^T G^{-1}(z_i-y_i)-\frac{N}{2}\ln|G| \quad (21)$$

where G is the covariance matrix of residuals.

During the solution the filter recursively processes measurement one at a time, and at each point produces the minimum variance state estimate based on all the data received up to that point. The ML estimation is a batch processor, therefore it uses the entire data record for each iteration. The estimates for the unknown parameters $\gamma$ (the coefficients in $\Gamma$ and filter gain are also included) and matrix G are found from equations (13) and (14) respectively, replacing $R_1$ by G. Once $\hat{\gamma}$ and $\hat{G}$ are obtained elements in $R_1$ and Q can be also estimated (6).

### 3.4 Comparison of methods

The output error methods give biased estimates when noise is present in measured state variables. The methods are also very sensitive to bias errors in measured data. Good fit exists only between measured and computed derivatives of state variables. The other disadvantage is the independence of each equation which is to be minimized. On the other hand the methods are very simple to apply both to linear and nonlinear systems and they can be used as a good start-up procedures for the iterative methods.

The WLS estimation provides unbiased estimates of unknown parameters provided that the mathematical model is correct and there is no noise in the measured inputs. The only significant cost is due to inaccuracy in the measured output variables.

For the ML technique it is assumed that the distribution of measured variables is known. There is no a priori knowledge regarding the values of unknown parameters $\gamma$. If the gaussian distribution is further assumed then the ML estimate is identical to the minimum variance estimate. The cost function for the ML technique is given as the negative likelihood function L(p).

In the Bayesian estimation technique the unknown parameters are treated as random variables. The optimum estimate is taken as the mean of the conditional distribution $p(\gamma/z)$. This result is reminiscent to the basic concept of the ML estimation. If the gaussian distribution for $p(z/\gamma)$ and $p(\gamma)$ is adopted then the Bayesian estimate can be treated as the minimum variance estimate with expanded cost function

$$J = \frac{1}{2}\sum_{i=1}^{N}(z_i-y_i)^T R_1^{-1}(z_i-y_i) + $$
$$+ \frac{1}{2}(\gamma-\gamma_o)^T R_2^{-1}(\gamma-\gamma_o) + \frac{N}{2}\ln|R_1| \quad (22)$$

which includes a penalty for departure from a priori values $\gamma_o$.

If the statistical data of a priori values are not available the Bayesian estimation can be simplified to the ML estimation with a priori weighting of some or all parameters. The matrix $R_2^{-1}$ in equation (18) is replaced by the weighting matrix taking into account the confidence on the a priori known parameters.

If no process noise is present the general ML estimation is reduced to the ML estimation and the residuals are the output errors. For the case where no measurement noise exists, the measurement noise covariance R1 is identically zero. If all the states and their derivatives are measured, then the likelihood function is the sum of squares of the equation errors at sampling times. Thus, the ML estimates are identical to the equation error estimates.

## 4. ACCURACY AND SENSITIVITY

Very often the accuracy of an estimation procedure is judged by deviations both in the parameters of the mathematical model and in the output variables. From the comparison of the estimation methods it follows that the ML estimation provides the most accurate estimate of unknown parameters. These estimates are consistent, asymptotically normal and efficient under very general conditions. The unknown parameters, $\gamma$, form a random vector with mean value $\hat{\gamma}$ and the lower bound on the covariance matrix $M^{-1}$.

The closeness of computed and measured outputs may be defined by a criterion developed from the cost function. For the ML estimation technique the term $\ln|R_1|$ can be, therefore, adopted as a measure for the fit error. The disadvantage of this criterion is that it improves monotonically with the increasing number of unknown parameters which could result in accurate fit but inaccurate parameter values due to identifiability problems.

The accuracy of estimated parameters is closely related to the sensitivity. The concept of sensitivity can be treated as the sensitivity of the function $y = g(x, u, \beta)$ with respect to the parameters. This approach means that the system is sensitive to changes in one particular parameter if a small change results in a large change in y. On the other hand, if y changes by only a small amount when a parameter is changed, y is insensitive with respect to that parameter and its estimation could result in the inaccurate value.

The numerical values of the sensitivities are included in the sensitivity and the information matrix. In the first case the coefficient in the jth row and kth column is formed by the sum of $\partial y_j (t_i)/\partial \gamma_k$ over the number of data points.

It is shown in (6) that the jth row and kth column in M is

$$M_{jk} = \sum_{i=1}^{N} (\frac{\partial v}{\partial \gamma_j})^T \bar{G}^1 (\frac{\partial v}{\partial \gamma_k}) + \frac{1}{2} T_r \left| \bar{G}^1 \frac{\partial G}{\partial \gamma_j} \bar{G}^1 \frac{\partial G}{\partial \gamma_k} \right| \quad (23)$$

For the ML estimation without process noise and measurement noise in the input variables equation (23) is simplified as

$$M_{jk} = \sum_{i=1}^{N} (\frac{\partial y}{\partial \gamma_j})^T \bar{R}_1^1 (\frac{\partial y}{\partial \gamma_k}) \quad (24)$$

## 5. IDENTIFIABILITY

The concept of identifiability is usually related to various problems connected with the ability to identify the parameters in the model assumed for the system. The first group of problems mentioned is referred to the identifiability of the parameters of the system. They can be solved by finding the maximum number of parameters, which can be identified from measurement of the system input - output data, and their location in the matrices (4).

The remaining problems are mostly connected with the identifiability of a description of the system which means the possibility of a numerical solution of the estimation process and the reliability of the estimates in terms of physically realistic values and small error covariances for the parameters. These identification problems are mainly influenced by the design of the experiment, the adequacy of the model, the number of unknown parameters and the sensitivity of output variables to these parameters.

The design of an experiment represents the wide range of characteristics, from which the input form is the most important one. The basic demand on the input is the proper excitation of all modes involved within the frequency range of interest. In addition it must not be possible to express the input as a linear combination of system response variables.

An efficient tool for improving the identifiability is the use of a priori known values of the parameters from theoretical predictions, wind-tunnel or previous flight measurement. Then the a priori values can be included in the estimation procedure as a set of fixed values, a set of values with weights expressing their uncertainty (the a priori weighting) and finally in the form of probability distribution (the Bayesian estimation).

The identifiability of a description of the system is discussed in (8) and (6) where other approaches towards the improvement of the identifiability are mentioned, too.

## 6. COMPUTING ALGORITHMS

Several computing algorithms for the estimation of aircraft parameters were published. The most general one is in (6) using the generalized ML estimation for a nonlinear system and including some ways for solving the identifiability problems.

The algorithm for the output error method and a linear model is in (11) together with the corresponding Fortran program. The ML estimation precedure combined with general equations of motion of an aircraft is developed in (12). Both of these programs were applied successfully to the analysis of flight data of many different types of aircraft.

A lot of effort has been devoted to the development of a computing algorithm and program at C.I.T. which would cover the equation of error and output error methods with various estimation techniques, which would be applicable to linear as well as nonlinear systems and which would be flexible enough for solving various identifiability problems.

The main problem in this algorithm was to cover both linear and nonlinear models with one set of expressions. It consisted in finding general forms for the constraint and sensitivity equations for both systems. The solution has been, in the main, found in the introduction of the augmented input vector which includes nonlinear terms in x, ux and u, as well as the input variables u (8).

The whole scheme was implemented on a small 16 bit word length computer using an intermediate level interpretation language developed in (13). This language offered considerable operational flexibility and made the program very suitable for the research in system identification.

## 7. FUTURE DEVELOPMENT

Considering the experience achieved and problems met in the process of aircraft identification, and assuming the future demands on the application of estimation techniques, the topics for the further research can be summarized as:-

1. extensive application of the generalized maximum likelihood technique, which considers the measurement noise in the input variables and the external disturbances, for the estimation of the aircraft states and parameters. This very general approach would cover the determination and analysis of aircraft performance, stability and control characteristics, and handling and riding qualities,

2. identification of aerodynamic derivatives from flight data for specific important flight regimes (e.g. high angle of attack and transonic flights, large disturbance combined manoeuvres) and unorthodox aircraft (STOL, VTOL) design. This will require the continuing development in methods for model structure determination and verification since the models in such aircraft are not well known,

3. correlation of the flight test results with wind-tunnel results for high performance aircraft and aircraft of special design,

4. development of methods for solving identifiability problems and the determination of the effect of a stability augmentation system and a human pilot on the identifiability of the parameters,

5. development of methods for the design of optimal inputs for linear as well as nonlinear systems. Modification of the optimal input for pilot acceptability and case of implementation,

and flight test validation of optimal inputs.

## 8. EXAMPLES

As examples, the parameter estimation of two different aircraft is presented. In both cases no process noise and no measurement noise in the input variables have been assumed. For the evaluation of parameters the ML estimation technique with or without a priori weighting has been used.

In the first example the data is taken from the measurement of slender delta-wing research aircraft in its longitudinal motion. The responses were excited from the horizontal steady-state flights at different airspeeds by the elevator deflection. Because of the input form used the airspeed changes during the transient motion were negligible. For the identification the time histories of the elevator angle, $\eta$, rate of pitch, $q$, and vertical acceleration, $n_z$, were available.

In the second example, parameters of a fighter aircraft are estimated. The lateral motion of the aircraft was excited from diving bank-turn flight using a short rudder pulse. The aileron and the elevator were used only in the compensatory role. The experiment was the preliminary one and the aircraft was not fully instrumented. Consequently the time histories of the rudder deflection, rate of pitch, bank and pitch angle were not recorded. This resulted in missing vital information on the primary input and in inadequate data for the steady-state conditions. The parameter estimation was based on the time histories of the aileron deflection, $\xi$, rate of roll, $p$, rate of yaw, $r$, sideslip vane, $\beta_V$, and lateral acceleration, $n_y$. The starting point for the analysis was selected at the time when the rudder was assumed to be returning to its zero deflection.

### 8.1 Slender Delta Wing Research Aircraft Longitudinal Dynamics

Taking into account the wind-tunnel test and preliminary flight test results, the perturbation

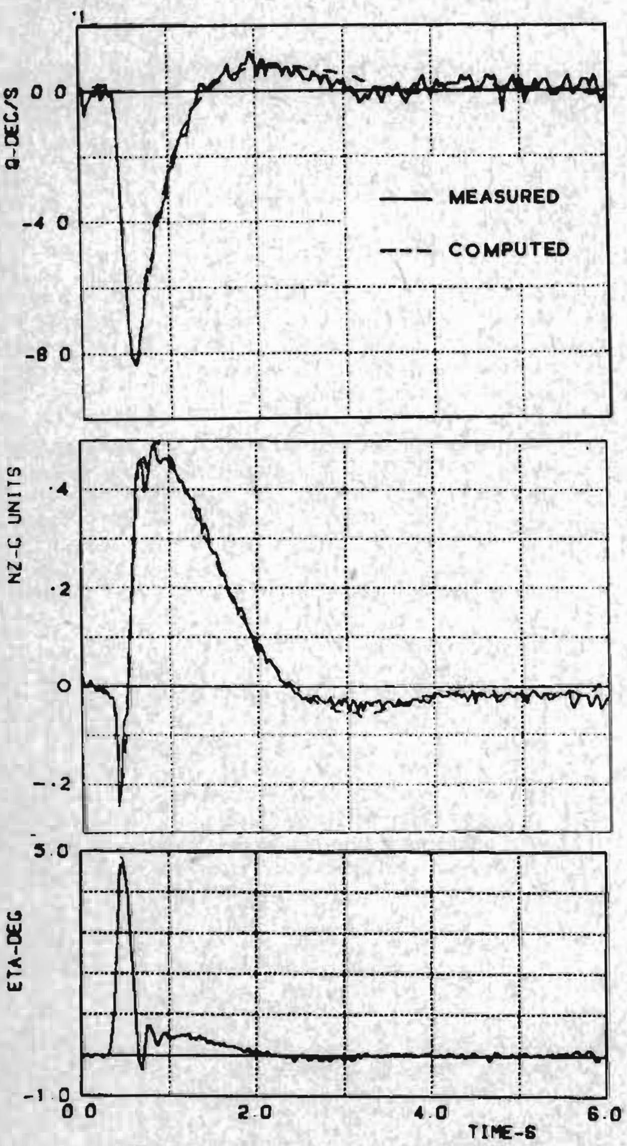


Fig.3 Measured and Computed Time Histories using ML estimation ($\alpha_e$ = 7.2 deg).

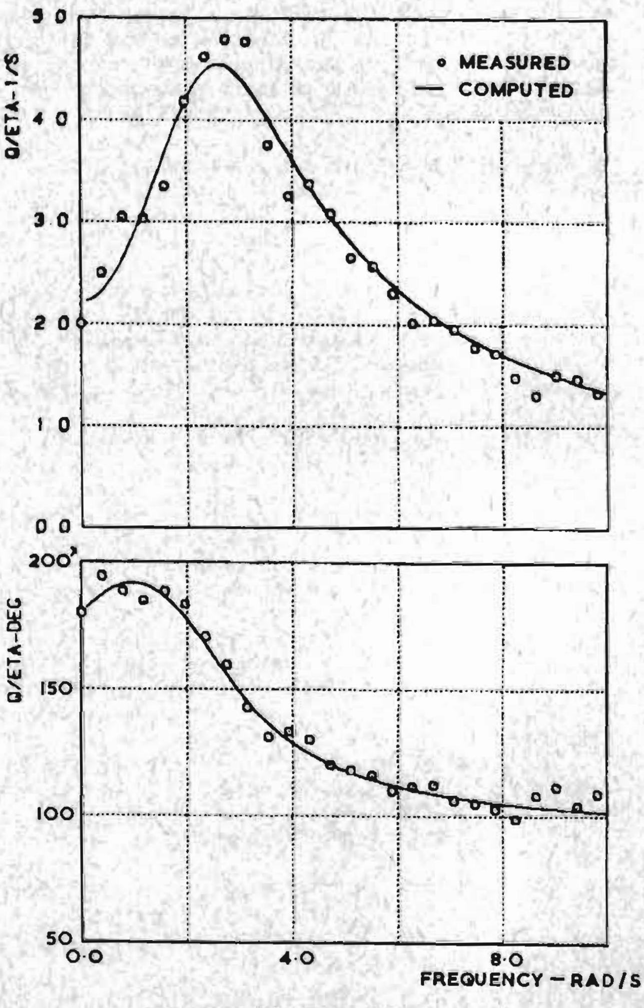


Fig.4 Measured and Computed Frequency Response Curves using ML estimation ($\alpha_e$ = 7.2 deg)

equations of motion are considered in the form

$$\dot{\alpha} = Z_\alpha \alpha + Z_q q + Z_{\alpha^2}\alpha^2 + Z_{q\alpha} q\alpha + Z_{\eta\alpha}\eta\alpha + Z_\eta \eta + Z_{\dot{\eta}}\dot{\eta} + Z_o$$

$$\dot{q} = M_\alpha \alpha + M_q q + M_{\alpha^2}\alpha^2 + M_{q\alpha} q\alpha + M_{\eta\alpha}\eta\alpha + M_\eta \eta + M_{\dot{\eta}}\dot{\eta} + M_o \tag{25}$$

$$\dot{\theta} = q$$

The vertical acceleration can then be expressed as

$$\frac{g}{u} n_z \equiv n_z^* = Z_\alpha \alpha + Z_q^1 q + Z_\theta \theta + Z_{\alpha^2}\alpha^2 + Z_{q\alpha} q\alpha + Z_{\eta\alpha}\eta\alpha + Z_\eta \eta + Z_{\dot{\eta}}\dot{\eta} + Z_o^1 \tag{26}$$

where u is the velocity, $\theta$ is the pitch angle, $\alpha$ is the angle of attack, and $Z_o$, $Z_o^1$ and $M_o$ are the bias terms. All remaining parameters are defined in (14).

For the parameter estimation, the linear model with $Z_{\dot{\eta}} = M_{\dot{\eta}} = 0$ was first used. The further simplification was achieved by fixing parameters $Z_q$ and $Z_q^1$ on their wind-tunnel values and parameter $Z_\eta$ on values from the steady-state flights. Comparison of measured and computer time histories at a low angle of attack is given in Fig. 3. The fit between the two output variables is very good. The resulting parameters and the lower bounds on their standard errors are presented in the third column of Table 1. The estimated parameters accord well with predictions.

The use of the linear model in the case of low angle of attack ($\alpha_e$ = 7.2 deg.) was also substantiated by comparing previous results with those from the ML estimation in the frequency domain, and the ML estimation with a priori weighting and the nonlinear model having $Z_{\eta\alpha} = M_{\eta\alpha} = 0$. The a priori values were taken from the linear case with weights proportional to their standard errors.

The values of estimated parameters are in the fourth and fifth column of Table 1, whereas the measured and computed frequency response curves are plotted in Fig. 4. The results of the two approaches mentioned did not produce significantly different values of parameters. The estimates of variance $s^2(n_z^*)$, however, can be considered as significantly different. The measure for significant difference in parameters is based on $2\sigma$ confidence intervals, and that for those in variances of measurement noise on the critical value for their ratio.

In the second test case of a high angle of

TABLE 1. Predicted and Estimated Parameters by ML Estimation with Lower Bounds on their Standard Errors.

| ITEM | $\alpha_e$ = 7.2 deg. | | | | $\alpha_e$ = 20.3 deg. | | |
|---|---|---|---|---|---|---|---|
| | PRED. | LINEAR MODEL | | NONLINEAR MODEL | PRED. | LINEAR MODEL | NONLINEAR MODEL |
| | | TIME DOMAIN | FREQ. DOMAIN | | | | |
| $Z_\alpha$ | - 1.74 | - 1.52 (0.03) | - 1.49 (0.06) | - 1.54 (0.03) | - 1.02 | - 1.43 (0.06) | - 1.48 (0.04) |
| $Z_{\alpha^2}$ | - 5 | - | - | - 2 (1) | - | - | - 7 (4) |
| $Z_{q\alpha}$ | - | - | - | - 0.5 (0.4) | - | - | - 0.9 (2) |
| $Z_{\eta\alpha}$ | - | - | - | - | - | - | - 0.6 (2) |
| $Z_\eta$ | - 0.38 | - | - | - 0.35 (0.01) | - 0.26 | - | - 0.23 (0.01) |
| $Z_{\dot{\eta}}$ | - | - | - | - 0.009 (0.001) | - | - | - 0.006 (0.002) |
| $Z_o$ | 0 | 0.0006 | - | 0.0020 | 0 | 0.020 | 0.020 |
| $Z_o^1$ | 0 | - 0.0003 | - | 0.0009 | 0 | 0.008 | 0.008 |
| $M_\alpha$ | - 6.85 | - 5.37 (0.9) | - 5.8 (0.2) | - 5.42 (0.09) | - 0.65 | - 1.0 *) (0.1) | - 0.97 (0.09) |
| $M_q$ | - 1.63 | - 1.99 (0.06) | - 1.7 (0.1) | - 2.21 (0.06) | - 1.1 | - 1.03 *) (0.07) | - 0.81 (0.06) |
| $M_{\alpha^2}$ | - | - | - | 3 (5) | - | - | - 73 (11) |
| $M_{q\alpha}$ | - | - | - | - 17 (3) | - | - | 50 (8) |
| $M_{\eta\alpha}$ | - | - | - | - | - | - | 178 (37) |
| $M_\eta$ | - 12.6 | - 14.1 (0.2) | -13.0 (0.4) | - 14.07 (0.02) | - 3.4 | - 3.22 (0.07) | - 3.23 (0.06) |
| $M_{\dot{\eta}}$ | - | - | - | - 0.08 (0.03) | - | - | - 0.17 (0.02) |
| $M_o$ | 0 | 0.013 | - | 0.018 | 0 | 0.030 | 0.034 |
| $s^2(q)$ | - | $2.3 \times 10^{-5}$ | - | $2.1 \times 10^{-5}$ | - | $2.3 \times 10^{-5}$ | $2.4 \times 10^{-5}$ |
| $s^2(n_z^*)$ | - | $4.8 \times 10^{-6}$ | - | $2.7 \times 10^{-6}$ | - | $19.5 \times 10^{-6}$ | $7.0 \times 10^{-6}$ |
| $\ln\lvert\hat{R}_1\rvert$ | - | - 22.9 | - | - 23.6 | - | - 21.5 | -22.5 |

*) strong correlation

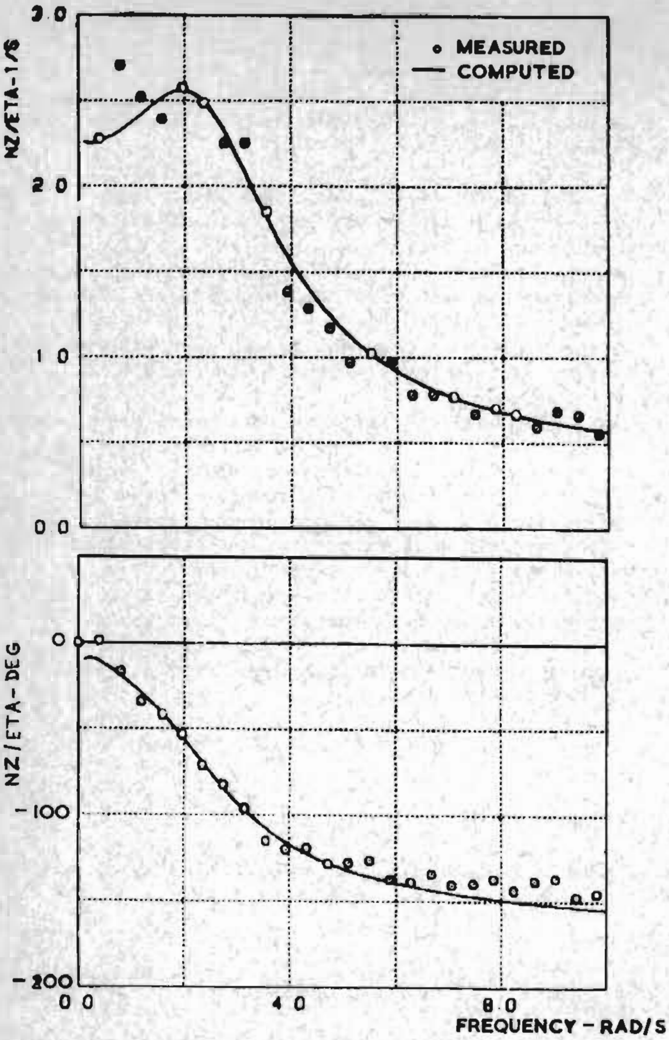


Fig.4 Measured and Computed Frequency Response Curves using ML estimation ($\alpha_e$ = 7.2 deg) - Concluded.

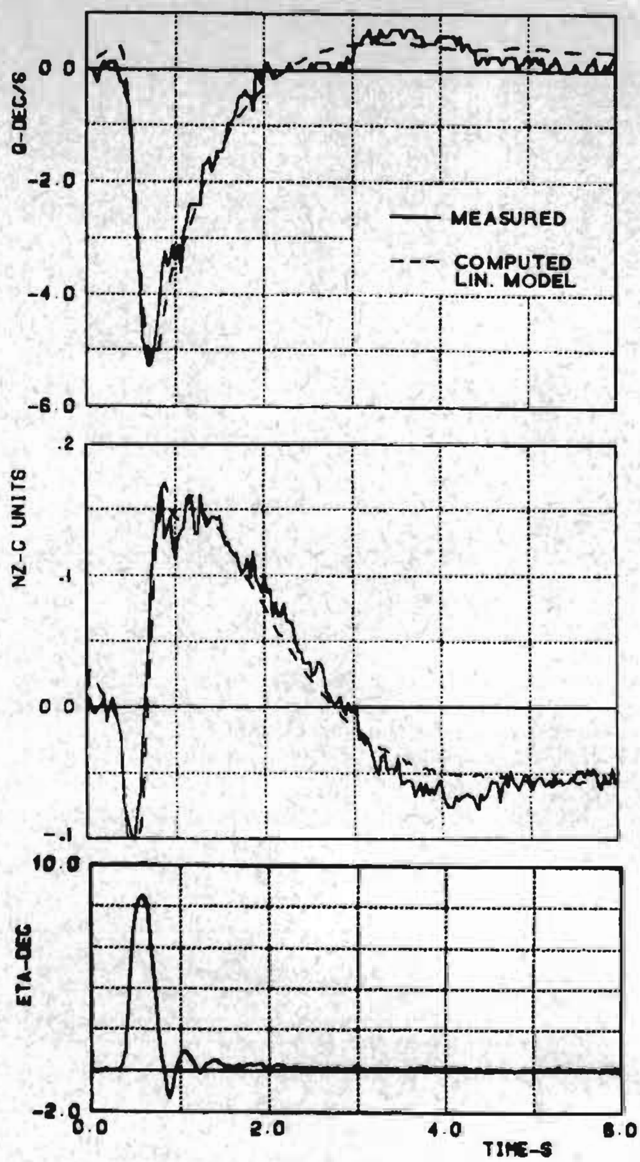


Fig.5 Measured and Computed Time Histories using ML estimation. Linear model, $\alpha_e$ = 20.3 deg.

attack ($\alpha_e$ = 20.3 deg), the linear model was proved to be inadequate. This follows from the comparison of the measured and computed output time histories presented in Fig. 5 and Fig. 6 for the linear model and the nonlinear model with a priori weighting. The substantial improvement of the fit in the vertical acceleration is apparent and is confirmed by the numerical values of corresponding variance estimates in Table 1.

In addition to these comparisons the time histories of residuals in the vertical acceleration and the corresponding autocovariance functions were computed and plotted as in Fig. 7. The autocovariance function for the nonlinear model is quite close to that assumed for the random white noise.

The use of the nonlinear model also had the significant effect on the damping parameter $M_q$ whereas the remaining parameters from the linear model and their counterparts in the nonlinear model do not differ significantly. These results are given in the second part of Table 1. The strong correlation between $M_\alpha$ and $M_q$ in the linear case was removed by the addition of nonlinear terms.

The importance of proper input form in this experiment is demonstrated on the estimates of the lift-curve slope parameter $Z_\alpha$ from the two test runs. The flight conditions for the two runs were the same but the applied pulse inputs differed in time length as shown in Fig. 8. The resulting harmonic content (Fourier transform) for the two inputs are also presented in Fig. 8.

The values of $Z_\alpha$ and coefficients in the sensitivity and information matrices related to this parameter are given in Table 2. The use of sharp pulse in Run 2 resulted in an inaccurate value of $Z_\alpha$ due to small excitation of nz and low sensitivities in the two output variables with respect to $Z_\alpha$.

### 8.2 Fighter Aircraft - Lateral Dynamics

The linearized equations of motion for a fighter aircraft are

$$\begin{bmatrix}\dot{p}\\ \dot{r}\\ \dot{\beta}\\ \dot{\phi}\end{bmatrix} = \begin{bmatrix} L_p & L_r & L_\beta & 0\\ N_p & N_r & N_\beta & 0\\ Y_p & Y_r & Y_\beta & Y_\phi\\ 1 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}p\\ r\\ \beta\\ \phi\end{bmatrix} + \begin{bmatrix} L_\xi & L_o\\ N_\xi & N_o\\ 0 & Y_o\\ 0 & 0\end{bmatrix}\begin{bmatrix}\xi\\ 1\end{bmatrix} \qquad (27)$$

The measurement equations are formulated as

$$\begin{bmatrix}p_E\\ r_E\\ \beta_{VE}\\ n^*_{yE}\end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & K_\beta\frac{x_v}{u} & K_\beta & 0\\ 0 & 0 & Y_\beta & 0\end{bmatrix}\begin{bmatrix}p\\ r\\ \beta\\ \phi\end{bmatrix} + \begin{bmatrix}p_e\\ r_e\\ \beta_{ve}\\ n^*_{ye}\end{bmatrix} + n \qquad (28)$$

where $\phi$ is the bank angle, $K\beta$ is the aerodynamic correction of sideslip vane, $x_v$ is the distance of sideslip vane with respect to aircraft c.g., and the indices E and e denote the measured and steady-state values respectively. The remaining parameters are explained in (8).

Assuming $Y_p$, $Y_r$, $Y_\phi$ and $x_v$ as known values the model includes 21 unknown parameters together with the four initial conditions $p_o$, $r_o$, $\beta_o$ and $\phi_o$. Because of the limited information contained in the measured data, the first estimate included only five stability parameters and three bias terms as unknown parameters. The remaining parameters were fixed on theoretical or wind-tunnel values, or on values estimated from recorded time histories. The resulting estimates are given in the first column of Table 3.

In the following estimation, additional unknown parameters have been included using the previous results for the a priori weighting. After several successive computing runs, all unknown parameters were evaluated at least once. Their final values are summarized in the last column of Table 3.

The substantial improvement in the fit of all outputs is apparent from the estimates of variances and logarithm of $|R_1|$. The measured time histories of the output variables from the last computing run are plotted in Fig. 9. The imperfection of the fit may be caused by a rudder motion not having

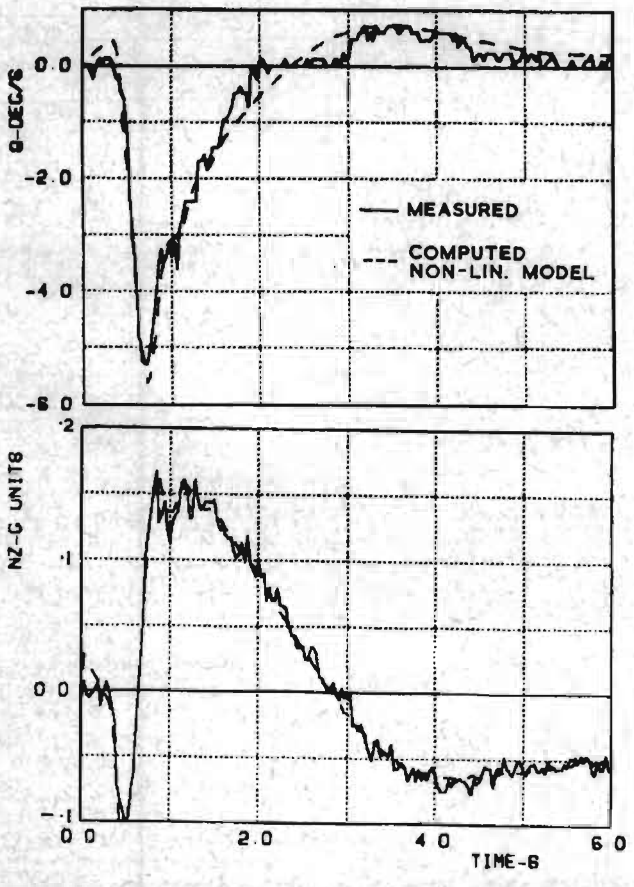


Fig.6 Measured and Computed Time Histories using ML estimation. Nonlinear model, $\alpha_e$ = 20.3 deg.

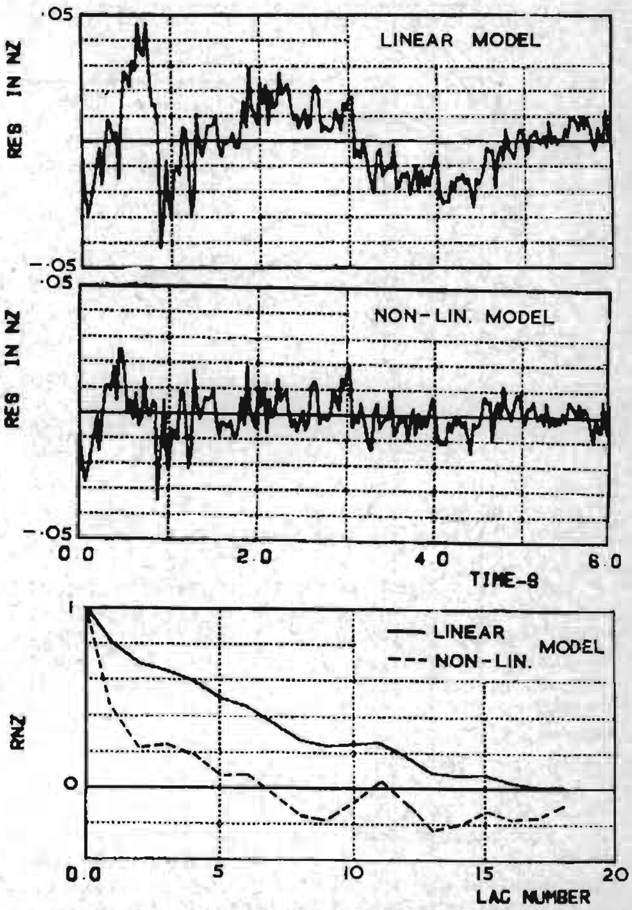


Fig.7 Time Histories of Residuals and Sample Autocovariance Functions of Residuals ($\alpha_e$ = 20.3 deg).

TABLE 2. Estimated Parameters and Sensitivities for two Different Inputs

| ITEM | RUN 1 | RUN 2 |
|---|---|---|
| $Z_\alpha$ | -1.29 (0.02) | -1.69 (0.06) |
| $\sum_{i=1}^{N} \partial q/\partial Z_\alpha$ | 1.276 | -0.0179 |
| $\sum_{i=1}^{N} \partial n_z^*/\partial Z_\alpha$ | -1.409 | -0.0126 |
| $M_{11}$ | 4909.5 | 449.9 |

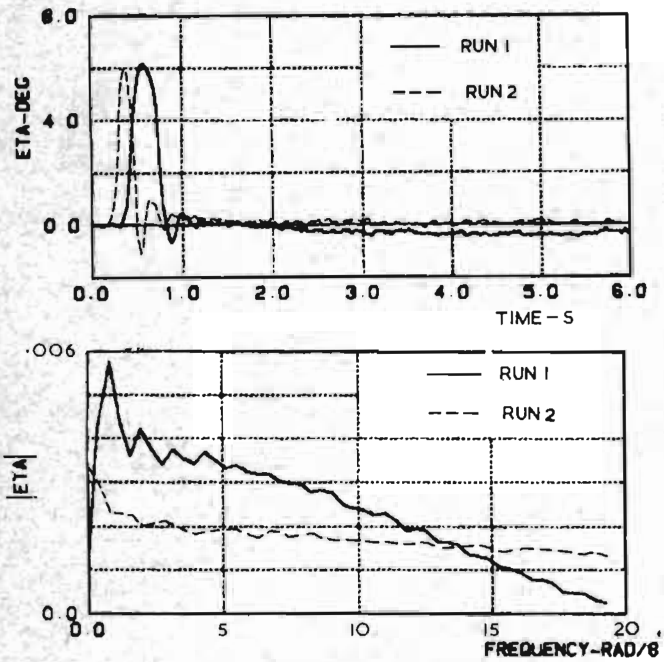


Fig.8 Different Input Forms used and their Harmonic Content.

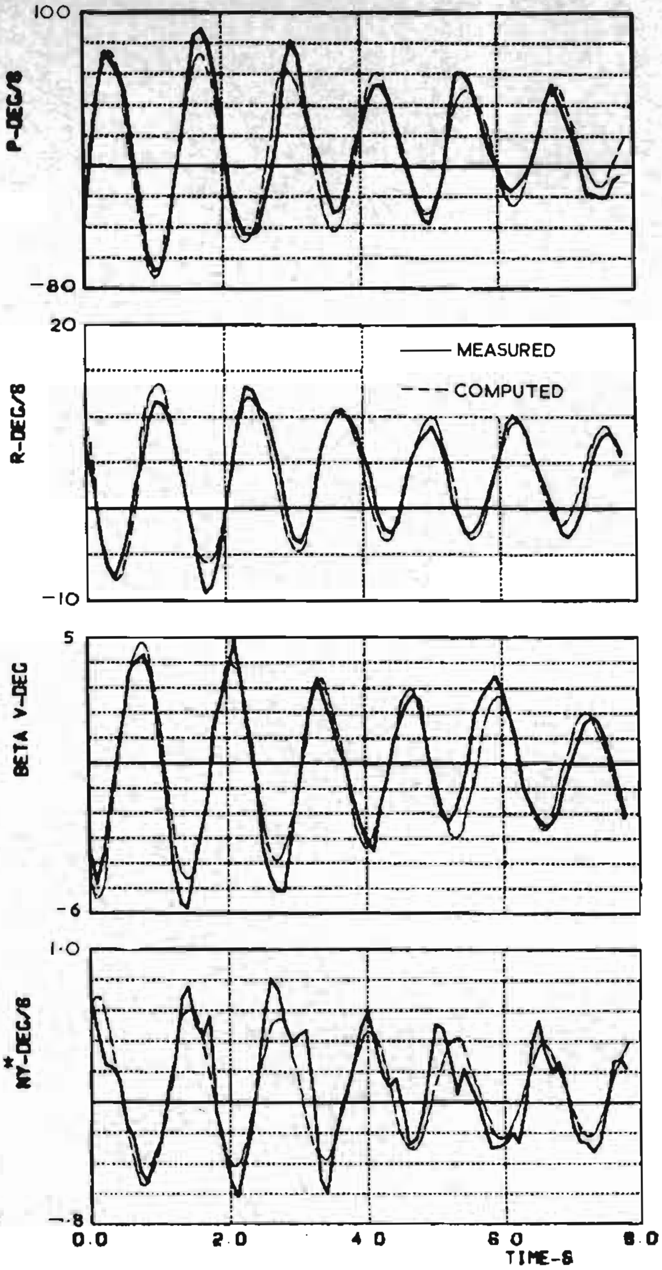


Fig.9 Measured and Computed Time Histories using ML estimation. Fighter Aircraft.

been accounted for and/or by a coupling of the lateral and longitudinal motion.

The test run presented was executed under buffet conditions (high angle of attack and high Mach number) for which the reliable estimates of stability derivatives were not available. It is, therefore, not possible to draw any serious conclusions from the comparison of estimated and predicted values.

## 9. CONCLUSION

The most advanced techniques for the evaluation of aerodynamic derivatives from flight test data are based on the identification of an aircraft. As the results of the identification aircraft parameters and their accuracies are estimated. In addition, the accuracy of the whole process can be evaluated and some idea about the correctness of the mathematical model for an aircraft can be found. The identification procedure includes three steps, namely characterization, parameter estimation and verification.

Characterization is a qualitative operation defining the structure of a model describing the motion of the aircraft under test. The problem is to select the simplest approximate representation that will permit the successful determination of the unknown parameters from measured data.

For parameter estimation three methods are usually used. The general maximum likelihood (ML) method is capable of solving the most general identification problem including the presence of additive random process noise in the equations of motion and random disturbances corrupting the

TABLE 3. Predicted and Estimated Parameters by ML Estimation with Lower Bound on their Standard Errors.

| ITEM | PRED. | FIRST ESTIMATES | | LAST ESTIMATES | |
|---|---|---|---|---|---|
| $L_p$ | - 1.9 | - 2.3 | (0.3) | - 1.9 | (0.1) |
| $L_r$ | - 1.7 | - | | - 4.6 | (0.7) |
| $L_\beta$ | - 8 | - 120 | (7) | - 121 | (1) |
| $N_p$ | 0.24 | - | | 0.068 | (0.005) |
| $N_r$ | - 0.31 | 0.8 | (0.1) | 0.619 | (0.008) |
| $N_\beta$ | 14 | 16.8 | (0.3) | 16.74 | (0.07) |
| $Y_\beta$ | - 0.20 | - 0.17 | (0.01) | - 0.182 | (0.009 |
| $L_o$ | - | 160 | (9) | 33 | (6) |
| $N_o$ | - | 13 | (1) | 3.2 | (0.8) |
| $Y_o$ | - | 2.6 | (0.4) | 1.3 | (0.3) |
| $K_\beta$ | 1 | - | | 1.53 | (0.06) |
| $p_o$ | - 36.9 | - | | - 38 | (3) |
| $r_o$ | 6.36 | - | | 6.1 | (0.3) |
| $\beta_o$ | - 3.06 | - | | - 3.0 | (0.1) |
| $\Phi_o$ | - 14.4 | - | | - 14 | (3) |
| $p_e$ | 11.4 | - | | 14 | (2) |
| $r_e$ | 0.63 | - | | 3.3 | (0.5) |
| $\beta_{ve}$ | 0 | - | | - 0.1 | (0.2) |
| $n^*_{ye}$ | 0.04 | - | | 0.07 | (0.03) |
| $s^2(p)$ | - | 517 | | 120 | |
| $s^2(r)$ | - | 5.1 | | 2.6 | |
| $s^2(\beta_v)$ | - | 1.0 | | 0.65 | |
| $s^2(n^*_y)$ | - | 0.031 | | 0.029 | |
| $\ell n\|R_1\|$ | - | 4.4 | | 1.8 | |

measured inputs and outputs. If no process noise is presented the general ML estimation is reduced to the output error method. For the case where no measured noise exists and all the states and their derivatives are measured the output error method is simplified to the equation error method.

Verification relates the results of parameter estimation to well known physical points of the system under investigation. If some inconsistency in the estimates and/or inadequacy in the mathematical model appears the whole problem of identification is eventually reconsidered at the characterization level.

Aircraft identification has been successfully used on many different types of aircraft. However further research in this area is needed if the procedure is to be used on a routine basis for the evaluation of aerodynamic derivatives from flight data and with further extension for the evaluation of aircraft performance.

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## D I S C U S S I O N

B.E. Karlin (Kiryat-Tivon, Israel): What methods do you use in order to filter the noise out of the measurement?

V. Klein: The measured data usually possesses high signal to noise ratio, therefore no pre-filtering is used prior to the application of any of the methods for system identification.

These methods work also as a filter and enable us to reconstruct the flight path and to estimate measurement and process noise characteristics.